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Signal and System 1st Summary

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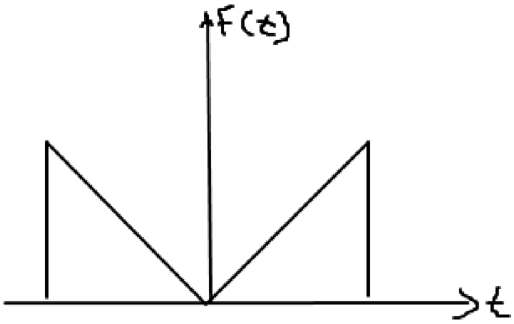
Even and Odd Signals

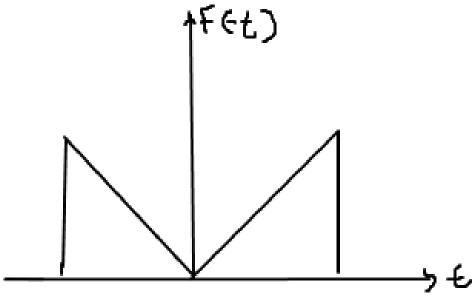
Even signal

Even signal are signals that remain identical under time reversal or reflection.

If the signal

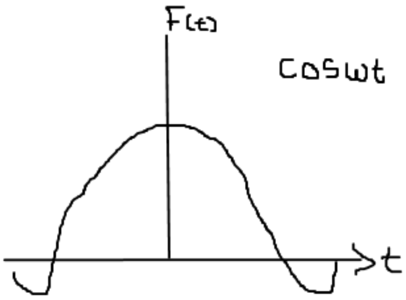
x(t) = x(-t) x(t) = x(-t) true for all values of time (t)

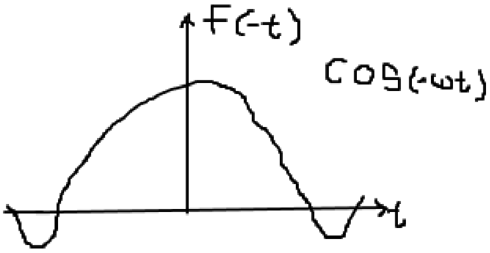




A very good example of an even signal is

x(t)=cos(wt) t=-t x(-t)=cos$(-wt)$





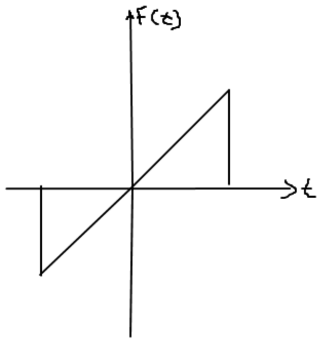
Even signals are symmetric about Y-axis

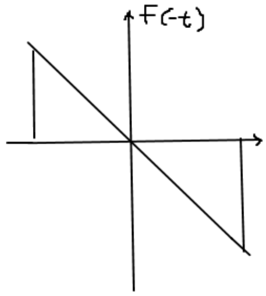
cos(-$\theta) = \cos\theta$ x(-t)=-cos$(wt) = x(t) = \cos(wt)$

<u>Odd signal</u>

Odd signals are signals that doesn't remain identical under time reversal or reflection of the signal

x(-t)=-x(t)



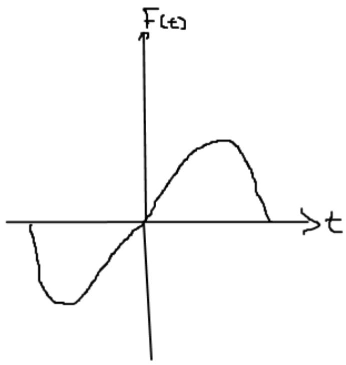


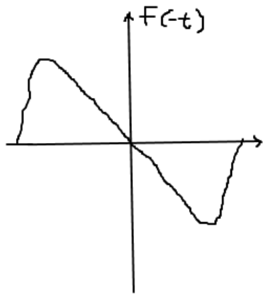
x(t) = -x(-t)

if t=0

x(0) = -x(-0)   x(0) = 0

Example of odd signals is   x(t) = $\sin \omega t$





x(-t) =-sin ωt

$\sin(-\theta) = -\sin\theta$ x(-t) = -sin ωt

Example1   Find the even and odd

x(t) = $e^{-2t}\cos t$

<u>Solution</u>

x(t) = $e^{-2t}\cos t$ x(t) = $e^{-2t(-t)}\cos(-t)$

$\cos x = \frac{e^{ix}+e^{-ix}}{2}$ and $sinx\,\frac{e^{ix}+e^{-ix}}{2i}$

x_e(t) = $\frac{1}{2}(x(t) + x(-t))$ Finding the Even components

x_e(t) $= \frac{1}{2}$(x(t) - x(t)) = x_e(t) $= \frac{1}{2}(e^{-2t}\cos t + e^{2t}\cos(t))$

x_e(t) = $\cos t(\frac{e^{-2t}+e^{2t}}{2})$

But $\cosh(x) = \frac{e^{-x}+e^{x}}{2}$ and $\sinh(x) = \frac{e^{x}-e^{-x}}{2}$

x_e(t) = cos(t)cosh(2t)

For Odd component

x_o(t) = $\frac{1}{2}(x_{(t)}$-$x_{(t)})$

$x_o(t) = \frac{1}{2}(e^{-2t}\cos t + e^{2t}\cos(t))$ = x_o(t) = $\cos t\,\frac{(e^{-2t}-e^{2t})}{2}$

$x_o(t)$ = -cos$(t)\sinh(2t)$

Example2. Find the even and odd component of the signal Bellow

x(t) = $-\cos(t) + \sin(t) + \cos(t)\sin(t)$

<u>Solution</u>

x(t) = $\cos t + \sin t + \cos t \sin t$

x(-t) = $\cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$

but $\sin(-\theta) = -\sin\theta$ & cos(-θ) $= \cos\theta$

x(-t) = $\cos(t) - \sin(t) - \cos(t)\sin(t)$

$x_o = \frac{1}{2}(x(t) + x(-t))$

$=\frac{1}{2}(\cos(t) + \sin(t) + \cos(t)\sin(t) + \cos(t) - \sin(t) - \cos(t)\sin(t))$

$=\frac{1}{2}$(2cos(t)) = $\cos(t)$

x_o(t) = $\frac{1}{2}$(x(t) - x(t))

$= \frac{1}{2}((\cos(t) + \sin(t) + \cos(t)\sin(t) - (\cos(t) - \sin(t) - \cos(t)\sin(t))$

$= \frac{1}{2}(\cos(t) + \sin(t) + \cos(t)\sin(t) - \cos(t) + \sin(t) + \cos(t)\sin(t)$

$= \frac{1}{2}(2\sin(t) + 2\cos(t)\sin(t))$

x_o(t) = $\sin(t) + \cos(t)\sin(t)$

Q2. Find the even and odd component

$$x(t) = t^2 \sin(t) - \frac{t^2}{sin^2(t)} + t^3 \cos(t) - \frac{cos^3(t)}{t^2} + \frac{t^5}{sin^5(t)}$$

<u>Solution</u>

$$x(t) = t^2 \sin t - \frac{t^2}{\sin t^2 t} + t^3 \cos t - \frac{cos^3 t}{t^2} + \frac{t^5}{sin^5 t}$$

N:B some basic rules O x O = E, E x E = E, O x E = O, E x O = O.

Taking the time reversal of the signal t = -t

x(-t) = $(-t)^2 \sin(-t) - \frac{(-t)^3}{sin^2(-t)} + (-t)^3 \cos(-t) - \frac{cos^3(-t)}{(-t^2)} + \frac{(-t^5)}{sin^5(-t)}$

$(-t)^2$ = -t x –t = t^2, sin(-t) = - sin(t), $(-t)^3$ = -t x –t x –t = $-t^3$ cos(-t) = cos(t)

$sin^2(-t) =$ sin(-t) x sin(-t) = $\sin^2$(t) because O x O = E

$\cos^3$(-t) = cos(-t) x cos(-t) x cos(-t) = $\cos^3$(t) because E x E x E = E, $(-t^5) = (-t^5)$

$sin^5(-t) =$ sin(-t) x sin(-t) x sin(-t) x sin(-t) x sin(-t) = $-\sin^5$(t)

Because O x O x O x O x O = E

x(-t) = $t^2(-\sin(t)) - \frac{(-t^3)}{sin^2 t} + (-t^3)\cos(t) - \frac{(cos^3(t))}{t^2} + \frac{(-t^5)}{(-sin^5(t))}$

x(-t) = $-t^2 \sin(t) + \frac{t^3}{sin^2 t} - t^3 \cos(t) - \frac{cos^3(t)}{t^2} + \frac{t^2}{sin^5(t)}$

Finding the Even signal $x_e = \frac{1}{2}(x(t) + x(-t))$

$$=\frac{1}{2}\Big((t^2\sin(t) - \frac{t^2}{sin^2(t)} + t^3\cos(t) - \frac{cos^3(t)}{t^2} + \frac{t^5}{sin^5(t)}) + (-t^2\sin(t) + \frac{t^3}{sin^2 t} - t^3\cos(t) - \frac{cos^3(t)}{t^2} + \frac{t^2}{sin^5(t)})\Big)$$

$$x_e(t) = \frac{1}{2}(-\frac{2\cos(t)}{t^2} + \frac{2t^2}{sin^5(t)})$$

$$x_e(t) = -\frac{\cos(t)}{t^2} + \frac{t^5}{sin^5(t)}$$

Now finding the Odd signal x_o(t) = $\frac{1}{2}$(x(t) - x(t))

$$= \frac{1}{2}\Big((t^2\sin(t) - \frac{t^2}{sin^2(t)} + t^3\cos(t) - \frac{cos^3(t)}{t^2} + \frac{t^5}{sin^5(t)}) - (-t^2\sin(t) + \frac{t^3}{sin^2 t} - t^3\cos(t) - \frac{cos^3(t)}{t^2} + \frac{t^2}{sin^5(t)})\Big)$$

$$x_o(t) = \frac{1}{2}(2t^2\sin(t) - \frac{2t^2}{sin^2(t)} + 2t^3\cos(t))$$

$$x_o(t) = t^2\sin(t) - \frac{t^2}{sin^2(t)} + t^3\cos(t)$$

PERIODIC AND NON-PERIODIC SIGNAL

A signal is said to be periodic if it repeat itself after a regular interval of time.

x(t) = x(t + T) $for\ all\ time\ t$

Example1.

Determine whether or not the following signals are periodic, for each signal that is periodic determine the fundamental period.

i. x(n) = $\cos(0.125\pi n)$

<u>Solution</u>

x(n) = $\cos(\frac{\pi}{8}n)$

x(n) = x(t + T) x(n) = $\cos(\frac{\pi n}{8})$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$, T$=\frac{1}{f} = \frac{1}{\frac{\omega}{2\pi}}$, $T = \frac{2\pi}{\omega} = 2\pi \div \frac{\pi}{8}$

T$= 2\pi \div \frac{8}{\pi}$

$T = 16sec$

x(n) = x(n+ T) x(n)= $\cos\left(\frac{\pi}{8}(n + 16)\right)$

x(n + T) = $\cos\left(\left(\frac{\pi}{8}n + \left(\left(\frac{\pi}{8}\right)16\right)\right)$

x(n + T) = $\cos\left(\frac{\pi}{8}n + 2\pi\right)$

$\text{Cos}(\theta + 2\pi) = Cos\theta$

x(n + T) = $\cos\left(\frac{\pi}{8}n\right)$

Since x(n + T) = x(n) therefore the signal is a periodic signal.

ii. $x(n) = R_e\left(e^{\frac{jn\pi}{12}}\right) + \ln\left(^{\frac{jn\pi}{12}}\right)$

<u>Solution</u>

$x(n) = \cos\left(\frac{n\pi}{12}\right) + i\,\sin\left(\frac{n\pi}{18}\right)$

$T_1 = \frac{2\pi}{\omega_1}$, $\omega_1 = \frac{\pi}{12}$ $T_1 = 2\pi \times \frac{12}{\pi} = 24sec$

$T_2 = \frac{2\pi}{\omega_2}$, $\omega_2 = \frac{\pi}{18}$ $T_2 = 2\pi \times \frac{18}{\pi} = 36sec$

T $= \frac{T_1 T_2}{\text{gcd}(T_1 T_2)} = \frac{24\ x\ 36}{\text{gcd}(24,\ 36)}$

Greatest common difference

T$= \frac{24 \times 36}{12} = 72$

iii. x(t) = $cos^2(2\pi t)$

Solution

x(t)= $cos^2(2\pi t)$

$cos^2\theta = \frac{1}{2}(1 + COS2\theta)$

$cos^2(2\pi t) = \frac{1}{2}(1 + COS(2 \times 2\pi t))$

$cos^2(2\pi t) = \frac{1}{2}(1 + COS4\pi t)$

$\omega = 4\pi$, $2\pi f = 4\pi$, $f = 2$, $T = \frac{1}{f} = 0.5$

x(t + T) $= x(t)$

x(t + 0.5) = $cos^2(2\pi$(t+0.5))

$= \frac{1}{2}(1 + COS4\pi(\text{t} + 0.5))$

$= \frac{1}{2}(1 + COS(4\pi\text{t} + 2\pi))$

$\text{Cos}(\theta + 2\pi) = Cos\theta$

$= \frac{1}{2}(1 + COS4\pi t)$

Since x(t + 0.5) $= x(t)$ Therefore the signal is periodic with time T = 0.5 sec

iv. x(t)= $e^{j\pi t}$

Solution

x(t)= $\cos \pi t - j \sin \pi t$

$\omega = \pi$, $\pi = 2\pi f$

$f = \frac{\pi}{2\pi} = 0.5$, $T = \frac{1}{f} = \frac{1}{0.5}$ = 2sec

x(t) = $(t + T)$ t = t + 2

$x(t+2) = \cos\pi(t+2) - j\sin\pi(t+2)$ $\quad$ $x(t+2) = \cos(\pi t + 2\pi) - j\sin(\pi t + 2\pi)$

$= (\cos\pi t\cos 2\pi + \sin 2\pi\sin\pi t) - j(\cos\pi t\sin 2\pi + \cos 2\pi\sin\pi t)$

$\text{Cos}(2\pi) = 1 \quad \text{Sin}(2\pi) = 0$

$= \cos\pi t - j\sin\pi t$

$X(t+T) = e^{j\pi t}$ since x(t) = x(t + T) therefore the signal is a periodic signal with T= 2

v. x(t) = $\cos(t + \frac{\pi}{4})$

<u>Solution</u>

x(t) = $\cos(t + \frac{\pi}{4})$

$= \cos(\omega t + \emptyset)$

$\omega = 1,\ 2\pi f = 1, \quad f = \frac{1}{2\pi},\ T = 2\pi$

$x(t + 2\pi) = \cos\left(t + 2\pi + \frac{\pi}{4}\right)$

$= \cos\left(t + \frac{\pi}{4} + 2\pi\right)$

$\cos(\theta + 2\pi) = cos\theta$

$X(t+T) = \cos\left(t + \frac{\pi}{4}\right)$

$X(t+T) = x(t)$

The signal is periodic with T= 2π sec

vi. x(t)= sin^2(t)

<u>Solution</u>

x(t)= $\frac{1}{2}(1 - cos2t)$

$\omega = 2, \quad 2\pi f = 2, \quad f = \frac{1}{\pi},\ T = \pi$

x(t)= $\frac{1}{2}(1 - cos2(t + \pi)$

x(t)= $\frac{1}{2}(1 - \cos(2t + 2\pi))$

$$COS(\theta + 2\pi) = COS\theta$$

x(t + T)= $\frac{1}{2}(1 - cos2t)$

x(t + T)= x(t)

The signal is a periodic signal with T=π

vii. x(t)= $e^{j\left(\frac{\pi}{2}t-1\right)}$

<u>Solution</u>

$x(t) = \cos\left(\frac{\pi}{2}t - 1\right) + jsin\left(\frac{\pi}{2}t - 1\right)$

$\omega = \frac{\pi}{2} = 2\pi f,\quad f = \frac{\pi}{2} \times \frac{1}{2\pi} = \frac{1}{4}$

T= 4 sec

x(t + T) = $\cos(\frac{\pi}{2}(t + 4) - 1) + jsin\,(\frac{\pi}{2}(t + 4) - 1)$

= $\cos(\frac{\pi}{2}t + 2\pi - 1) + jsin(\frac{\pi}{2}t + 2\pi - 1)$

$$COS(\theta + 2\pi) = COS\theta$$

=$\cos(\frac{\pi}{2}t - 1) + jsin\left(\frac{\pi}{2}t - 1\right)$

x(t + T) $= e^{j((\frac{\pi}{2}t-1))}$

viii. x(n)= $\sin(\pi + 0.2\pi)$

<u>Solution</u>

x(n)= $\sin(0.2\pi + \pi)$

=$\sin(\,\omega t + \emptyset)$

$\omega = 0.2 = 2\pi f,\quad f = \frac{0.2}{2\pi} = \frac{2}{10} \times \frac{1}{2\pi},\quad = f = \frac{1}{10\pi},\quad T = 10\pi$

x(n + T) = $\sin(0.2(n + 10\pi) + \pi)$

$=\sin(0.2n + 2\pi + \pi)$

$$\text{Sin}(\theta + 2\pi) = sin\theta$$

$= \sin(0.2n + \pi + 2\pi)$

$= \sin(0.2n + \pi)$

Since x(t + 10π) $= x(t)$ Therefore the signal is periodic with time T = 10π sec

Summary by BADIRU SULAIMON

Reference

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