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Signal and System 2nd Summary

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ENERGY AND POWER SIGNAL

A signal is said to be Energy or Power signal if the total Power or Energy transmitted is Finite.

Energy signal

A signal is said to be an Energy signal when the Energy value of the signal is Finite

$$0 < E < \infty$$

The power of an Energy signal is zero $Therefore\ P = 0$

$$E = \int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$E_{\lim T\to\infty} = \int_{-T}^{T} x(t)^2 dt \qquad for\ infinite\ duration$$

Power signal

A signal is said to be a Power signal when the Power value of the signal is Finite

$$0 < P < \infty$$

The Energy of a power signal is infinity $Therefore\ E = \infty$

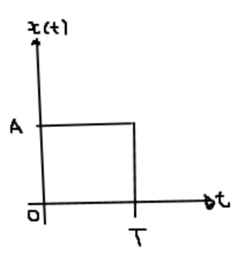
$$P = \frac{1}{2T}\int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$P_{\lim T\to\infty} = \frac{1}{2T}\int_{-T}^{T} x(t)^2 dt \qquad for\ infinite\ duration$$

If a signal those not certify any of the condition above it is sad to **be Neither Energy Nor Power Signal**

Question1.

Check whether the signal below is an energy or power signal



Solution

The limit of the signal is

$$x(t) = \begin{cases} A & t > 0 \\ 0 & otherwise \end{cases}$$

Since the duration is Finite $E = \int_{-T}^{T} x(t)^2 dt \quad for\ finite\ duration$

$$E = \int_0^T A^2 dt \quad = (A^2 t) \begin{matrix} T \\ 0 \end{matrix}$$

$$= (A^2 t - A^2 \times 0)$$

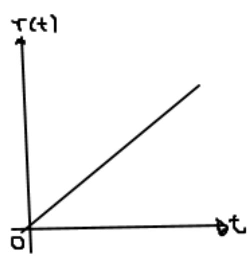
$$E = A^2 t \ \ Unit\ Joules$$

Since (A) can be any meaningful value therefore the signal is an Energy signal and the power is Zero.

Question2.

Check whether a ramp signal is an Energy or power signal

Solution



$$r(t) = \begin{cases} t, & 0 < t > \infty \\ 0, & otherwise \end{cases}$$

Since the duration is Finite

$$E = \int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$E = \int_0^\infty t^2 dt \quad = (\frac{t^3}{3}) \begin{matrix} \infty \\ 0 \end{matrix}$$

$$E = \left(\frac{\infty^3}{3} - \frac{0^3}{3}\right) = \frac{\infty^3}{3} = \infty$$

Since the Energy is infinity we have to check for the power

$$P = \frac{1}{2T}\int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$P = \frac{1}{2T}\int_{0}^{\infty} t^2 dt \quad = \left(\frac{t^3}{3}\right)_{0}^{\infty}$$

$$P = \frac{1}{2T}\left(\frac{\infty^3}{3} - \frac{0^3}{3}\right) = \frac{1}{2T} \times \frac{\infty^3}{3} = \infty$$

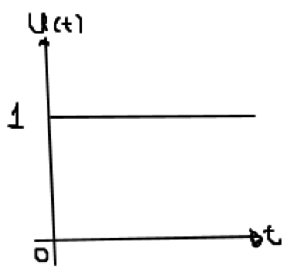
$$P = \infty\ and\ E = \infty\ \ the\ signal\ is\ neither\ Energy\ or\ Power$$

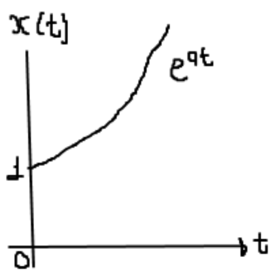
Question3.

Check whether the signal below is an Energy or power signal

$$x(t) = u(t)e^{at}$$

<u>Solution</u>





We get x(t) by multiplying thee unit step signal with e^{at}

Obtaining limit from x(t)

$$x(t) = \begin{cases} e^{at}, 0 < x < \infty \\ 0, \qquad otherwise \end{cases}$$

Since the duration is Finite

$$E = \int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$E = \int_0^\infty (e^{at})^2 dt \quad = \int_0^\infty e^{2at} dt \quad = (\frac{e^{2at}}{2a})\begin{matrix}\infty \\ 0\end{matrix}$$

$$E = \left(\frac{e^{2a\times\infty}}{2a} - \frac{e^{2a\times 0}}{2a}\right) = \left(\frac{e^{\infty}}{2a} - \frac{e^{0}}{2a}\right) = \left(\frac{\infty}{2a} - \frac{1}{2a}\right) = \infty - \frac{1}{2a} = \infty$$

NB : $e^{\infty} = \infty \;\; and \; e^{-\infty} = 0$

Since the Energy is infinity we have to check for the power

$$P = \frac{1}{2T}\int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$P = \frac{1}{2T}\int_0^\infty (e^{at})^2 dt \quad = \frac{1}{2T}\int_0^\infty e^{2at} dt \quad = \frac{1}{2T}(\frac{e^{2at}}{2a})\begin{matrix}\infty \\ 0\end{matrix}$$

$$P = \frac{1}{2T}\left(\frac{e^{2a\times\infty}}{2a} - \frac{e^{2a\times 0}}{2a}\right) = \frac{1}{2T}\left(\frac{e^{\infty}}{2a} - \frac{e^{0}}{2a}\right)$$

$$\lim T \to \infty$$

$$P = \frac{1}{2\times\infty}\left(\frac{\infty}{2a} - \frac{1}{2a}\right) = \frac{1}{\infty}\left(\infty - \frac{1}{2a}\right) = \frac{\infty}{\infty} = 0$$

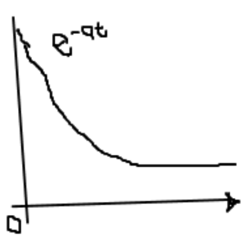
$$P = 0\ and\ E = \infty \;\; the\ signal\ is\ neither\ Energy\ or\ Power$$

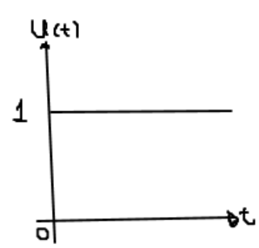
Question4.

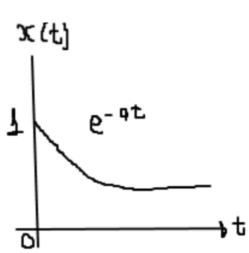
Check whether the signal below is an Energy or power signal

$$x(t) = u(t)e^{-at}$$

<u>Solution</u>







We get x(t) by multiplying thee unit step signal with e^{-at}

Obtaining limit from x(t)

$$x(t) = \begin{cases} e^{-at}, 0 < x < \infty \\ 0, \qquad otherwise \end{cases}$$

$$E = \int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$E = \int_{0}^{\infty} (e^{-at})^2 dt \quad = \int_{0}^{\infty} e^{-2at} dt \quad = (\frac{e^{-2at}}{-2a})\begin{matrix} \infty \\ 0 \end{matrix}$$

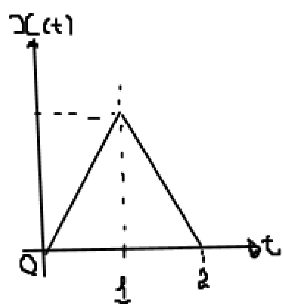
$$E = -\left(\frac{e^{-2a\times\infty}}{2a} - \frac{e^{-2a\times 0}}{2a}\right) = -\left(\frac{e^{-\infty}}{2a} - \frac{e^{0}}{2a}\right) = -\left(\frac{0}{2a} - \frac{1}{2a}\right) = -\left(0 - \frac{1}{2a}\right) = \frac{1}{2a}$$

$$E = \frac{1}{2a}\ Unit\ Joules$$

Since (a) can be any meaningful value therefore the signal is an Energy signal and the power is Zero.

Question5.

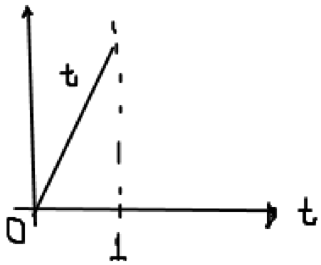
Check whether the signal below is an Energy or power signal



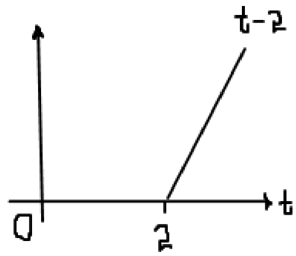
Solution

The signal is a combination of two Ramp signal (one ramp and another time scaled and reversed ramp)

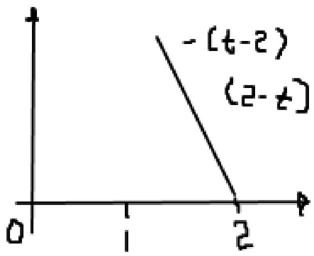
Splitting the signal into two different part we have



Normal ramp



A ramp delayed by $t_o = 2$



Reversing the signal

$$f(x) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t & 1 \le t \le 2 \\ 0, & otherwise \end{cases}$$

$$E = \int_{-T}^{T} x(t)^2 dt \qquad for\ finite\ duration$$

$$E = \int_0^1 (t)^2 dt \ + \int_1^2 (2-t)^2 dt$$

$$E = \int_0^1 (t)^2 dt \ + \int_1^2 (2-t)(2-t) dt \ = \int_0^1 (t)^2 dt \ + \int_1^2 (t^2 - 4t + 4) dt$$

$$\left(\frac{t^3}{3}\right)\begin{matrix}1\\0\end{matrix} + \left(\frac{t^3}{3} - \frac{4t^2}{2} + 4t\right)\begin{matrix}2\\1\end{matrix}$$

$$= \left(\frac{1^3}{3} - \frac{0^3}{3}\right) + \left(\left(\frac{2^3}{3} - \frac{4 \times 2^2}{2} + 4 \times 2\right) - \left(\frac{1^3}{3} - \frac{4 \times 1^2}{2} + 4 \times 1\right)\right)$$

$$\left(\frac{1}{3} + \frac{8}{3} - 8 + 8\right) - \left(\frac{1}{3} - 2 + 4\right) = \frac{2}{3} J$$

Operations on signals

Time Scaling

Y(t) is obtained by scaling the independent variable time (t) by a factor a

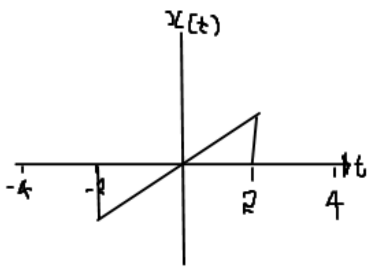
$$y(t) = x(at)$$

$$a > 1 \quad Compression\ of\ signal \quad i.e\ if\ a\ is\ any\ number\ greater\ than\ 1.$$

$$a < 1 \quad Expansion\ of\ signal \quad i.e\ a\ are\ set\ of\ number\ between\ 0\ and\ 1.$$

Example1.

For the signal below draw the signal operation



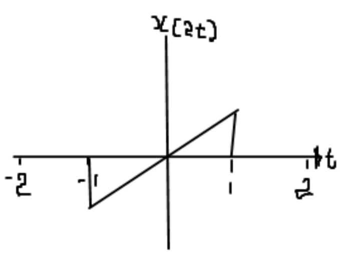
a. $x(2t)$ b. $x(\frac{1}{2}t)$

<u>Solution</u>

a. $x(2t)$ $\quad a = 2 \quad a > 1 \quad i.e\ the\ signal\ will\ be\ compressed$

$$New\ interval = -\frac{t}{a}\ and \frac{t}{a} = -\frac{2}{2}\ and \frac{2}{2}$$

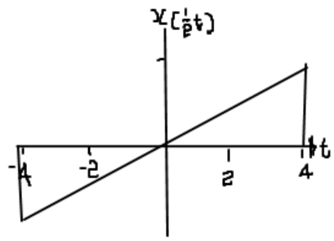
$$New\ interval = -1\ and\ 1$$



b. $x(\frac{1}{2}t)$ $\quad a = \frac{1}{2} \quad a < 1 \quad i.e\ the\ signal\ will\ be\ expanding$

$$New\ interval = -\frac{t}{a}\ and \frac{t}{a} = -2 \div \frac{1}{2}\ and\ 2 \div \frac{1}{2}$$

$$New\ interval = -2 \times 2\ and\ 2 \times 2 = -4\ and\ 4$$



Time Shift

A time shifted version of $x(t)$ is define as

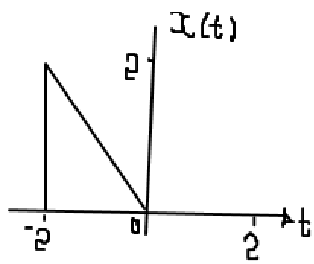
$$y(t) = x(t - t_o)$$
$$for\ t_o > 0\ i.e\ for + ve\ t_o \dots \dots \dots ..Delay(right\ shift)$$
$$for\ t_o < 0\ i.e\ for - ve\ t_o \dots \dots \dots ..Advance(left\ shift)$$

Time shift for a reversal signal $x(-t)$ is define as

$$y(t) = x(-t - t_o)$$
$$for\ t_o > 0\ i.e\ for + ve\ t_o \dots \dots \dots ..Avance(left\ shift)$$
$$for\ t_o < 0\ i.e\ for - ve\ t_o \dots \dots \dots ..Delay(right\ shift)$$

Example2.

Draw the following time shift from the signal below

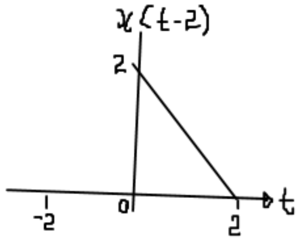


a. $x(t-2)$ b. $x(t+2)$ c. $x(-t+2)$

<u>Solution</u>

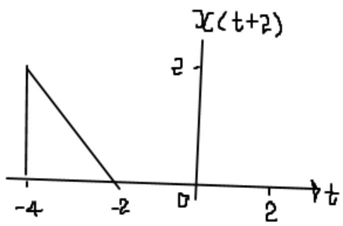
a. $x(t-2)$

$$t_o = 2\ \ which\ is\ positive, therefore\ it\ a\ Delay(right\ shift)$$



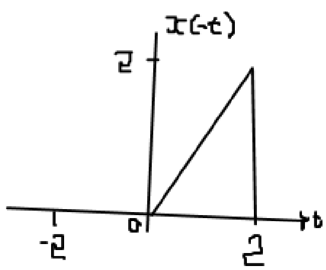
b. $x(t+2)$

$t_o = -2 \quad which\ is - ve, therefore\ it\ an\ advancement\ (left\ shift)$

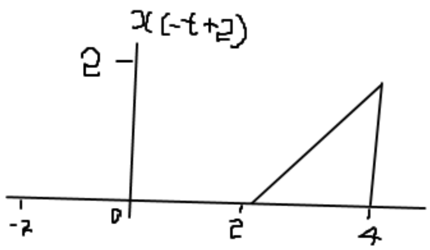


c. $x(-t+2)$

This is a shift of a reversal signal, Therefore



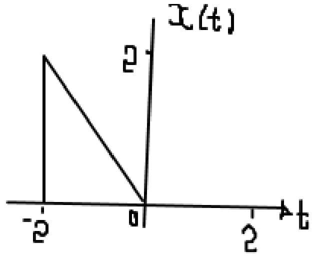
The reversal signal



$t_o = -2 \quad which\ is - ve, therefore\ it\ a\ delay\ (right\ shift)$ for a shifted reversal signal

Question1.

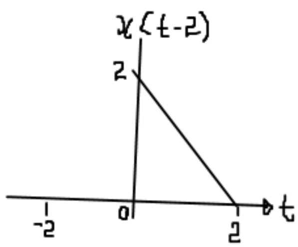
Draw the following output signal from $x(t)$



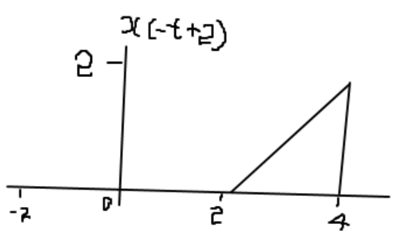
a. $y(t) = x(t-2) + x(-t+2)$
b. $y(t) = x(t-2) + x(-t-2)$
c. $y(t) = 2x(\frac{t}{2} - 2)$

<u>Solution</u>

a. $y(t) = x(t-2) + x(-t+2)$

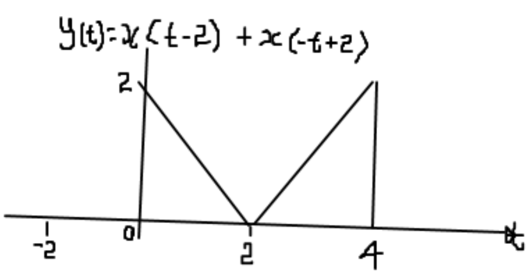


$$t_o = 2 \quad which\ is\ positive, therefore\ it\ a\ Delay(right\ shift)$$

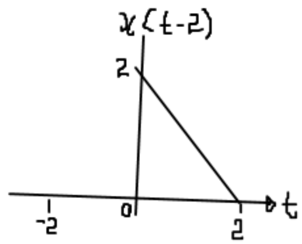


The sifted reversal signal

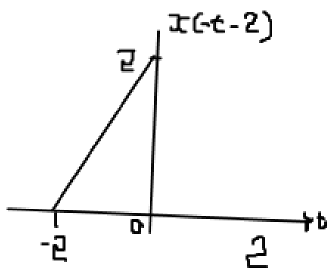
Adding the two signal we have;



b. $y(t) = x(t-2) + x(-t-2)$

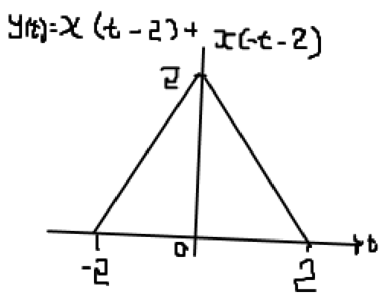


$$t_o = 2 \quad which\ is\ positive, therefore\ it\ a\ Delay(right\ shift)$$



$t_o = 2 \quad which\ is + ve, therefore\ it\ an\ advancement\ (left\ shift)$ For a shifted reversal signal

Adding the two signal we have;



c. $y(t) = 2x(\frac{t}{2} - 2)$

This operation is the combination of Time scaling, Time Shift and Amplitude Scaling

Time scaling, Time Shift and Amplitude Scaling Operation

Steps in performing such operations

$$if\ \ y(t) = bx(at - t_o)$$
$$where\ b = Amplitude\ scaling \quad a = Time\ scaling \quad t_o = Time\ shift$$

i. Neglect both the Amplitude and Time scaling and deal with time shift

$$v(t) = x(t - t_o)$$

ii. Scale the time of the shifted signal $v(t)$

$$s(t) = v(at)$$

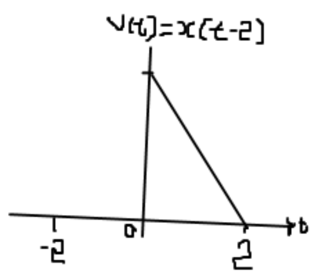
iii. Lastly scale the Signal $s(t)$

$$y(t) = bs(t)$$

$$y(t) = 2x(\frac{t}{2} - 2)$$

i. Neglecting the Amplitude and time scaling

$$v(t) = x(t - 2)$$



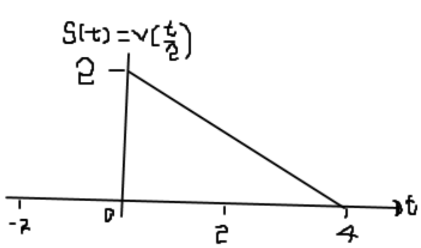
ii. Scale the time of the shifted signal $v(t)$

$$s(t) = v(\frac{1}{2}t)$$

$v(\frac{1}{2}t)$ $a = \frac{1}{2}$ $a < 1$ $i.e\ the\ signal\ will\ be\ expanding$

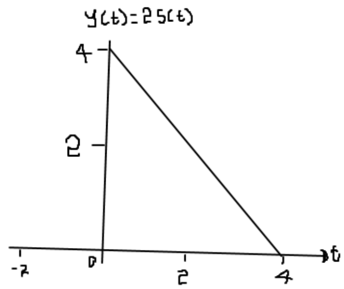
$$New\ interval = -\frac{t}{a}\ and \frac{t}{a} = 0 \div \frac{1}{2}\ and\ 2 \div \frac{1}{2}$$

$$New\ interval = \ 0 \times 2\ and\ 2 \times 2 = \ 0\ and\ 4$$



iv. Lastly scale the Signal $s(t)$

$$y(t) = 2s(t)$$



Next Topic Convolution

Summary by BADIRU SULAIMON

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