MTH1302 1st Summary

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Two Dimensional Coordinate Geometry

Coordinate geometry is a method of analyzing geometric shapes. In this tutorial our discussion will be limited to straight lines and circles.

Straight Line

What is a straight line?

A straight line is a line separating two point with zero curvature.

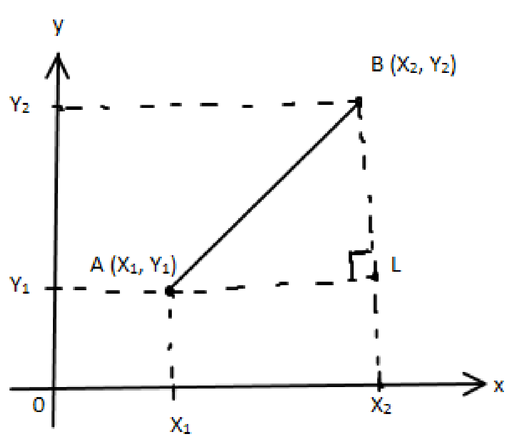
There are three types of point's position reference which are:

- Fixed Point (X, Y)
- Generic Points (X_1, Y_1) (X_2, Y_2)
- General Points(X, Y) , this point can be located anywhere or always varying

Analysis of a Straight Line

We analyze a straight line by calculating the following parameters depending on the type of question

a. Distance between two Points



Pythagoras theorem can be used in calculating the distance between two points A and B after forming a right angle triangle with the Straight Line.

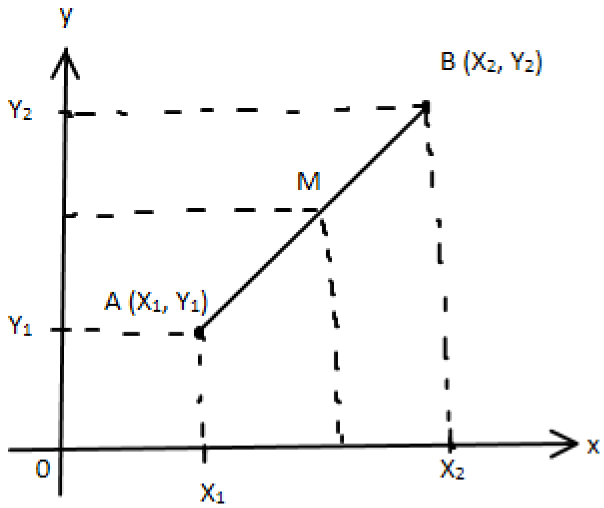
$$(AB)^2 = (AL)^2 + (BL)^2$$

$$(AB)^2 = (X_2 - X_1)^2 + (Y_2 - Y_1)^2$$

Where $AL = X_2 - X_1$ and $BL = Y_2 - Y_1$

$$(AB) = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

b. **Mid-point** : The midpoint is the center of a straight line



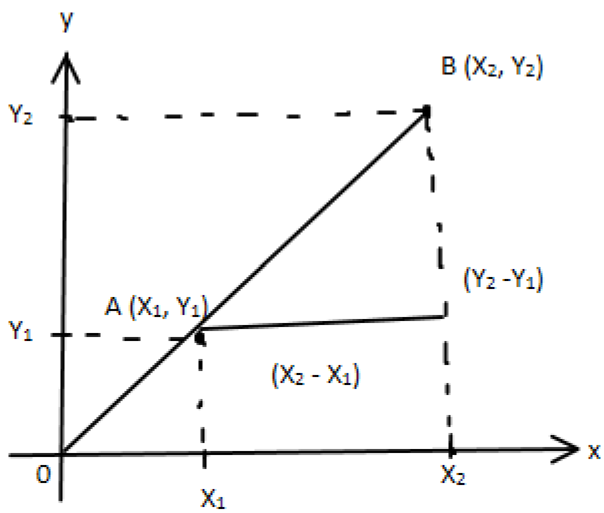
The center of X-Coordinate of M is given as

$$\frac{X_2 - X_1}{2}$$

The center of Y-Coordinate of M is given as

$$\frac{Y_2 - Y_1}{2}$$

c. **Gradient:** This is the inclination of the line towards the axis of the horizontal plane (X-axis).



The gradient of line $\overline{AB}$

$$\overline{AB} = \frac{Y_2 - Y_1}{X_2 - X_1} = m$$

NB

- Slope of two parallel line $m_1 and\ m_2$ are the same $m_1 = m_2$
- Slope of two perpendicular lines $m_1 and\ m_2$ is equal to -1
  $m_1 m_2 = -1, \quad m_1 = -\frac{1}{m_2}, \qquad m_2 = -\frac{1}{m_1}$

d. Equation of a straight Line

The general equation is given as:

$$\boldsymbol{y = mx + c}$$

Where:

$$\boldsymbol{m}\ is\ the\ slope\ m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boldsymbol{c}\ is\ the\ point\ on\ intercept\ on\ y - axis$$

NB: $(y - y_1) = m(x - x_1)$ can be used to find the equation of a straight line

<u>Examples</u>

1. Find the equation of a straight line through the point (-1, 3) with slope of 2

   <u>Solution</u>

   (x, y) (-1, 3) m=2

   $(y - y_1) = m(x - x_1) + c$

   $(y - 3) = 2(x - (-1)) + 0$

   $(y - 3) = 2(x + 1)$

   $y - 3 = 2x + 2$

   $y = 2x + 2 + 3$

   $y = 2x + 5$

2. Find the equation of a line through the point (1, 2) and (3, 1). What is its slope? What is the intercept

Solution

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-1}{1-3} = \frac{1}{-2} \qquad m = -\frac{1}{2}$$

$$(y - y_1) = m(x - x_1)$$
$$(y - 2) = \frac{-1}{2}(x - 1)$$
$$y - 2 = \frac{-1}{2}x + \frac{1}{2}$$
$$y = \frac{-1}{2}x + \frac{1}{2} + 2$$
$$y = \frac{-1}{2}x + \frac{5}{2}$$

$$y = mx + c \qquad slope = -\frac{1}{2} \quad and\ intercept = \frac{5}{2}$$

3. What is the equation of the line that passes through the point $(1,1)$ and perpendicular to the line $y = -2x + 2$, where did the two lines intercept?

Solution

a. $slope \qquad parallel\ \ lines \quad m_1 = m_2\ \ , perpendicular\ lines \quad m_1 m_2 = -1$

$$m_1 = -\frac{1}{m_2}$$

$$y = -2x + 2$$
$$m_1 = -2 \qquad m_2 = \frac{1}{m_1} = -\frac{1}{-2} = \frac{1}{2}$$

$therefore\ equation\ of\ \ (1,1)$

$$(y - y_1) = m(x - x_1)$$

$$(y - 1) = \frac{1}{2}(x - 1)$$
$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$
$$y = \frac{1}{2}x - \frac{1}{2} + 1$$
$$y = \frac{1}{2}x + \frac{1}{2} \qquad m = 1 \qquad c_1 = 2$$

$$m = \frac{1}{2} \qquad c_2 = \frac{1}{2}$$

b. Point of intercept

$y = -2x + 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\, eqn(1)$

$y = \frac{1}{2}x + \frac{1}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...\, eqn(2)$

$$\begin{array}{rl} & y = -2x + 2 \\ - & y = \frac{1}{2}x + \frac{1}{2} \\ \hline & 0 = -\frac{5}{2}x + \frac{3}{2} \end{array}$$

$-\frac{5}{2}x + \frac{3}{2}$ $\qquad \frac{5}{2}x = \frac{3}{2}$

$x = \frac{3}{2} \div \frac{5}{2} = \frac{3}{2} \times \frac{2}{5} = \frac{3}{5}$ $\qquad x = \frac{3}{5}$

$Subtituting\ x\ in\ \ eqn(1)$

$y = -2x + 2$

$y = -2\left(+2\left(\frac{3}{5}\right) + 2\right)$

$y = \frac{4}{5}$

$The\ point\ of\ intercept\ of\ the\ two\ lines\ is\ \ (\frac{3}{5}, \frac{4}{5})$

Exercise

1. Find the equation of the line connecting
   a. A(2, 6) and B(5, 8) $\qquad ans \quad 3y = 2x + 4$
   b. A(-1, 3) and B(11, 12)
2. The equation of a line $y = -2x + 3$ which is perpendicular to a line passing through (5,6)
   a. Find the equation of the line (5,6)
   b. Where do the two lines intersect
3. The point Q and P are coordinate (-1, 6) and (9, 0) respectively. The line L is perpendicular to PQ and passes through the mid-point of PQ. Find the equation of the line L
4. The equation of a line (l_1) is $2y - 3x - k = 0$ where k is a constant, given that the point A(1,4) lies on l_1 find

a. The value of K?
b. The gradient of l_1

Circle

What is a circle?

A circle is a simple shape of Euclidean Geometry consisting of those points in a plane which have an equal distance from the center.

What is a circle diameter?

Diameter is a straight line passing from side to side through the center of a circle. Unit is measured in meters (m)

What is circle radius?

This is simply half of the diameter of a circle

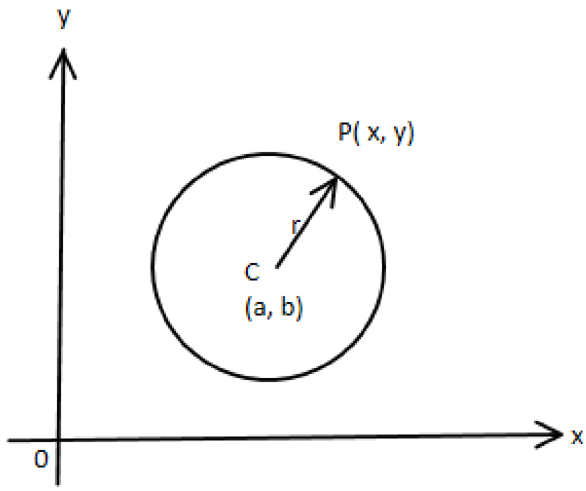
$$r = \frac{d}{2} \quad unit\ in\ meters\ (m)$$

Analysis of a Circle

The equation of a circle can be analyze depending on the question, below are some analyses of a circle.

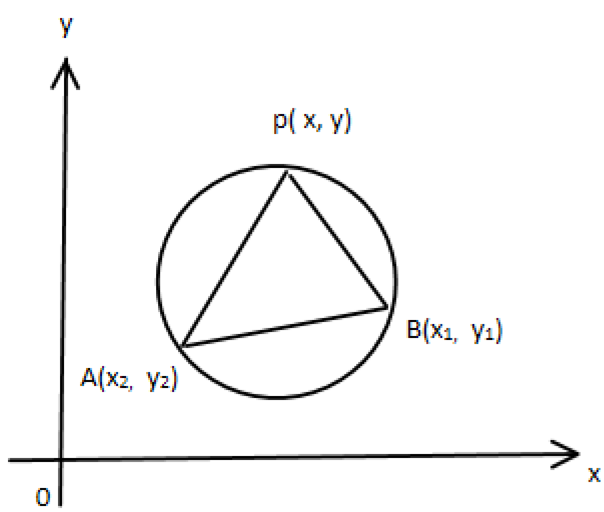
a. The general equation of a circle

This can also be analyze using Pythagorean Theorem



$$r^2 = (x - a)^2 + (y - b)^2$$

b. The equation of a circle whose diameter is joining two points A and B respectively can be (X_1, Y_1) and (X_2, Y_2) as shown below



$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2)$$

c. The equation of a circle, through three different points:

Assuming three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) then we have:

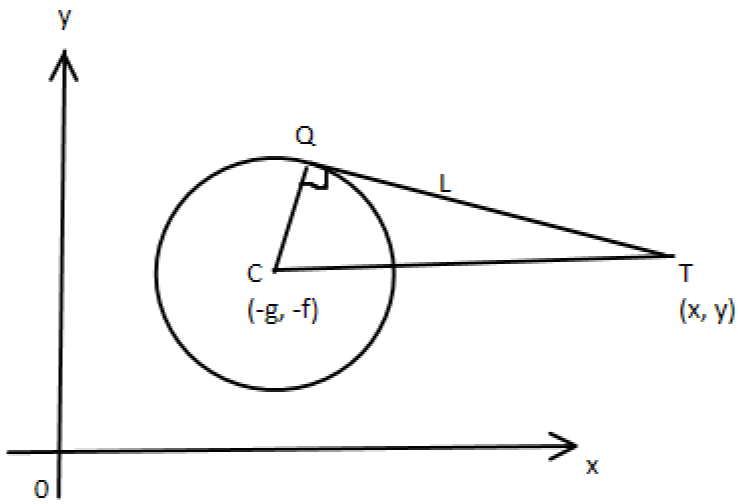
$$(x_1)^2 + (y_1)^2 + (2gx_1) + (2fy_1) + c = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$(x_2)^2 + (y_2)^2 + (2gx_2) + (2fy_2) + c = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

$$(x_3)^2 + (y_3)^2 + (2gx_3) + (2fy_3) + c = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

The constants g, f and c are found by solving the equations.

d. The equation of the length of the tangent from a given external points to a circle



Using Pythagoras Theorem $(TQ)^2 = (TC)^2 + (CQ)^2$

Where: T is the given point at (x, y)

TQ is the tangent to the circle

C is the center of the circle

$$Therefore: (TQ)^2 = (x)^2 + (y)^2 + 2gx + 2fy + c$$

Examples

Q1. Find the equation of a circle whose center is at point (-2, 3) and diameter has a length of 20

Solution

Equation of circle $(x - a)^2 + (y - b)^2 = r^2$

$d = 20\,,\ r = \frac{4}{5} = 10$

$a = -2, \quad b = 3$

$\left(x-(-2)\right)^2+(y-3)^2=10^2$

$(x+2)^2+(y-3)^2=100$

Q2. Find the center and the radius of the circle whose equation is given by $x^2+4x+y^2-8y=5$

<u>Solution</u>

$x^2+4x+y^2-8y=5$

Adding and subtracting half $(\frac{1}{2})$ of the coeeficient of x^2 and y^2

Coefficient of $x=4$ $\left(\frac{4}{2}\right)^2=2^2=4$

Coefficient of $y=8$ $\left(\frac{8}{2}\right)^2=4^2=16$

$x^2+4x+y^2-8y=5$

Adding and subtracting 4 and 16 to the equation

$x^2+4x+4-4+y^2-8y+16-16=5$

$x^2+2x+2x+4-4+y^2-4y-4y+16-16=5$

$x(x+2)+2(x+2)-4+y(y-4)-4(y-4)-16=5$

$(x+2)(x+2)-4+(y-4)(y-4)-16=5$

$(x+2)^2+(y-4)^2-20=5$

$(x+2)^2+(y-4)^2=25$ $c(-2,4)$

$(x-h)^2+(y-k)^2=r^2$ $r=5$

$-a=2$ $-b=-4$ $r^2=25$

$a=-2$ $b=4$ $r=\sqrt{25}=5$

Q3. Find an equation of the circle whose center is at the point $(-4,6)$ and passes through the point $(1,2)$

<u>Solution</u>

$c\ (-4,6)$ $p=(1,2)$

Finding the radius from

$(x-a)^2+(y-b)^2=r^2$

$c\ (a,b)$ $p=(x,y)$ $x=1$ $y=2$

$a=-4$ $b=6$

$r^2=\left(1-(-4)\right)^2+(2-6)^2$

$r^2 = \ (1+4)^2 + (-4)^2$

$r^2 = 25 + 16$

$r^2 = 41$

Equation of the line is

$(x+4)^2 + (y-6)^2 = 41$

Q4. Find an equation of the circle whose diameter d has endpoint at $(-5, 2)$ and $(3,6)$

<u>Solution</u>

The formula f a distance between two lines

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3-(-5))^2 + (6-2)^2}$$

$$AB = \sqrt{8^2 + 4^2} = \sqrt{80}$$

$$diameter = \sqrt{80}$$

$$radius = \frac{\sqrt{80}}{2} = \frac{\sqrt{16 \times 5}}{2} = \frac{4\sqrt{5}}{2} = 2\sqrt{5}$$

To find $c(x, y)$ we are going to apply the Midpoint rule

$$Midpoint\ rule \rightarrow \quad x = \frac{(x_1 + x_2)}{2} \qquad y = \frac{(y_1 + y_2)}{2}$$

$$Midpoint\ rule \rightarrow \quad x = \frac{(-5+3)}{2} = -\frac{2}{2} = -1 \qquad y = \frac{(2+6)}{2} = \frac{8}{2} = 4$$

Equation of the Line is

$$(x-a)^2 + (x-b)^2 = r^2$$

$$(x-(-1))^2 + (x-4)^2 = (2\sqrt{5})^2$$

$$(x+1)^2 + (x-4)^2 = 4 \times 5$$

$$(x+1)^2 + (x-4)^2 = 20$$

Q5. Find the X and Y intercepts of the graph of the circle given the equation $x^2 + 3x + y^2 - 4y = 18$

<u>Solution</u>

$$x^2 + 3x + y^2 - 4y = 18$$

$$Setting \quad y = 0$$

$$x^2 + 3x = 18 \qquad\qquad x^2 + 3x - 18 = 0$$

$$x^2 - 3x + 6x - 18 = 0 \qquad x(x-3) + 6(x-3) = 0$$

$$(x+6)(x-3) = 0$$

$$x = -6 \; and \; 3$$

$$Setting \quad x = 0$$

$$y^2 - 4y = 18 \qquad y^2 - 4y - 18 = 0$$

$$using\ all\ mighty\ formula\ \frac{(-b) \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1 \quad b = -4 \quad c = -18$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times -18}}{2 \times 1} = \frac{4 \pm \sqrt{16 + 72}}{2} = \frac{4 \pm \sqrt{88}}{2} = \frac{4 \pm \sqrt{4 \times 22}}{2}$$

$$= \frac{4 \pm 2\sqrt{22}}{2} = \frac{4 + 2\sqrt{22}}{2} \quad and \quad = \frac{4 - 2\sqrt{22}}{2}$$

$$y = 2 + \sqrt{22} \quad and \quad 2 - \sqrt{22}$$

The intercept at y $\left(0, 2 - \sqrt{22}\right)$ and $(0,\ 2 + \sqrt{22}\,)$ intercept at x $(-6, 0)$ and $(0, 3)$

<u>Exercise</u>

1. Find the point of interception of the circle with the equation $(x-2)^2 + (y-6)^2 = 40$ and the line with equation $y = 3x$

   Ans $(0,0)$ $(4, 12)$
2. Find the equation of the circle whose center is at the point $(2, -5)$ and is tangent to the X-axis

   Ans $(x-2)^2 + (y+5)^2 = 25$
3. Find the equation of a circle that is tangent to both x and y axes, with a radius of 4 whose center is located in the second quadrant

   Ans $(x+4)^2 + (y-4)^2 = 16$

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