PHY1230 1st Summary

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Elasticity

Elasticity: is the ability of a substance to regain its original shape and size after being distorted (stretched, bent, twisted or compressed) by an external force.

Elastic material: is one that regains its original shape and size after the distortion force has been removed.

Hooke's law

Hooke's Law state that, provided that the elastic limit of elastic material has not being exceeded, then force is directly proportional to the extension. When a sample is stretched this means that the extension of the sample increases linearly with the amount of force applied. Materials that obey Hooke’s law are called Hookean materials.

Hooke’s law can be written as; $F \alpha\ e$

$$F = Ke$$

$Where:$ $F = force$

$e = The\ extension$

$K = Strifness\ constant\ of\ the\ sample$

The limit of proportionality where there is no longer a linear relationship between Force and extension after more Force is applied the elastic limit will be reached. This means that the sample will no longer return to its original shape.

Solved problems

1. A helical spring (helix shape like spring) is stretched 2cm by a load of 6N. calculate
   a) The force constant
   b) The energy in the stretched spring.

Solution

a) $F = Ke$

e=2cm=0.02m, F=6N.

$$K = \frac{F}{e} = \frac{6}{0.02} = 300N/m$$

b) $E = {}^{1}/_{2}\, Ke^2 = {}^{1}/_{2} \times 300 \times (0.02)^2 = 0.06J$

2. A force of 2N stretches an elastic material by 30mm. what additional force will stretch the material 35mm? Assuming Hooke's law is obeyed.

Solution

$F = Ke$, F=2N, e=30mm=0.03m (convert from mm to m)

$2 = K \times 0.03$

$$K = \frac{2}{0.03} = 66.6N/m$$

Let the force stretching the material 35mm be F. since the elastic limit has not been exceeded the K is constant throughout that region. Therefore K=66.6N/m
New extension e=0.035m

$F = K \times 0.035 = 66.6 \times 0.035 = 2.33N$

<u>Mechanical properties of materials</u>

1. **Strength**: this is how great an applied force on a material can withstand before breaking.
2. **Stiffness:** is the opposition (resistance) a material set up to being distorted by having its shape or size or both changed. A stiff material is not flexible.
3. **Ductility/Workability**: is the ability of a material to be hammered, pressed, bend, rolled, cut or stretched into useful shapes.
4. **Toughness:** is a material that does not crack easily. (it is not brittle).
5. **Malleability**: is the ability of a material to be hammered or squeezed into their sheets. E.g Ag, Au, Sn, Cu and Al.

Tensile stress: this is the force acting per unit cross-sectional area.

Tensile strain: this is the extension produced per unit length when a material undergoes stress.

$$Tensile\ stress\ (\delta) = \frac{F}{A}$$

$$Tensile\ strain\ (\varepsilon) = \frac{e}{L}$$

For a material to obey Hooke's law, tensile stress is directly proportional to tensile strain:

Tensile stress α tensile strain.

Therefore tensile stress = K (tensile strain).

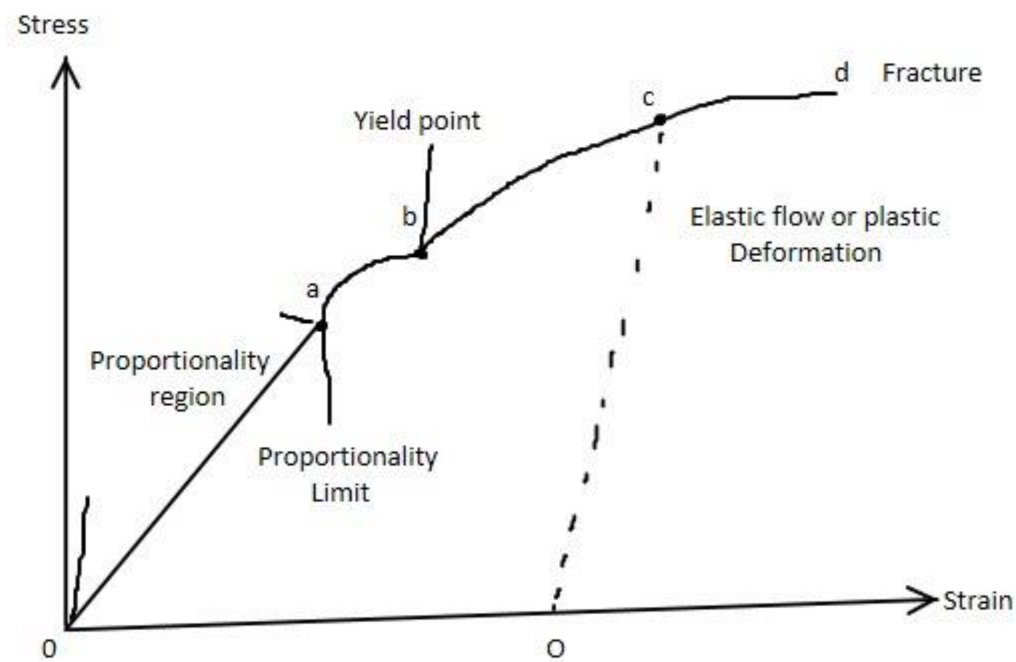

Figure 1; A graph of stress against strain.

The constant of proportionality K is called **ELASTIC MODULUS** of a material for the type of strain or deformation under consideration.

The moduli can be classified into three, one for each kind of strain.

1. Young modulus (E)
2. Modulus of rigidity or shear modulus (G)
3. Bulk modulus (B)

Young Modulus (E)

Young modulus (E) is the measure of a solid stiffness or resistance to elastic deformation under load. It relates stress (Force per unit area) to strain, thus a low young modulus value means a solid elastic or shift.

The equation for Yong's Modulus

$$E = \frac{\delta}{\varepsilon} = \frac{\left(\frac{F}{A}\right)}{\frac{\Delta L}{L_o}} = \frac{FL_o}{AL}$$

$$Where \quad E \text{ is the young modulus usually expressed in Pascal in } (N/m^2)$$

$$\delta = \text{the uniaxial Stress}$$

$$\varepsilon = \text{the strain}$$

$$F = \text{the Force of compression and extension}$$

$$A = \text{Cross section Area}$$

$$\Delta L(e) = \text{Change in lenght}$$

$$L_o(l) = \text{the original Length}$$

Solved problems

1. A metal bar of length 1.0m and uniform cross-sectional area $10^{-5}m^2$ extends by 1.5mm when a force of 200N is applied. If the elastic limit is not exceeded, calculate

a) The tensile stress

b) The tensile strain

c) The young modulus for the metal.

Solution

F=200N, e=1.5mm, L=1.0m=1000mm, $A = 10^{-5}m^2$.

a) Tensile stress $= \frac{F}{A} = \frac{200}{10^{-5}} = 2 \times 10^7 N/m^2$.

b) Tensile strain $= \frac{e}{L} = \frac{1.5}{1000} = 0.0015.$

c) Young modulus $= \frac{tensile\ stress}{tensile\ strain} = \frac{2\times10^7}{0.0015} = 1.3 \times 10^{10} N/m^2.$

2. A coil spring is used to support a 1.8 kg mass. If the spring stretches to 2cm, what is the spring constant? What mass would be required to stretch the spring to 5cm?

Solution

Data given: mass=1.80kg, e_1=2cm=0.02m, e_2=5cm=0.05m.

$$F = mg = 1.8 \times 9.8 = 17.64N$$

From Hooke's law, $F = Ke$

$$K = \frac{F}{e} = \frac{17.64}{0.02} = 882N/m.$$

If e_2=5cm,

$$F = Ke = 882 \times 0.05 = 44.1N.$$

Hence, F=mg

$$m = \frac{F}{g} = \frac{44.1}{9.8} = 4.50kg.$$

3. A wire whose cross-section is 4mm^2 is stretched by 0.1mm by a certain weight. How far will a wire of the same material and length stretch if its cross-sectional area is 8mm^2 and the same weight is attached?

Solution

For the first wire, A_1 =4mm^2, e_1=0.01

$$E_1 = \frac{F_1 L_1}{A_1 e_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For the second wire, $A_2=8mm^2$, $e_2=?$

$$E_2 = \frac{F_2 L_2}{A_2 e_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Since the second wire is of the same material and the same length with the first.

This implies that; $E_1=E_2$.

Hence, dividing equation 91) by (2)

$$A_1e_1=A_2e_2$$

$$e_2 = \frac{A_1 e_1}{A_2} = \frac{4 \times 0.1}{8} = 0.05mm$$

<u>Bulk modulus (B)</u>

Bulk stress: is the increase in force per unit area

Bulk strain: is the change of volume per original volume.

The bulk modulus (B) of a substance measure the substance resistance to isolate compression, it is the ratio of the infintesimal pressure increase to resulting relative decrease in the volume. The bulk modulus can be formally defined by the above equation.

$$K = -v \times \frac{dp}{dv}.$$

The negative sign is introduced to make B positive since Δv being a decrease is negative

Solved examples

1. Determine the fractional change in volume as the pressure of the atmosphere 0.1Kpa around a metal block is reduced to zero by placing the block in a vacuum. The bulk modulus for the metal is 12500Kpa.

Solution

$$\Delta p = 0.0 - 0.1 = -0.1 \times 10^3 Pa,$$
$$B = 125000 \times 10^3 Pa$$

$$B = \frac{-\Delta p}{\frac{\Delta v}{v}}$$

$$\frac{\Delta v}{v} = \frac{-\Delta p}{B} = \frac{-(-0.1 \times 10^3)}{1.25 \times 10^8} = 8 \times 10^{-7}$$

2. Compute the volume change of a solid copper cube, 40mm on each edge, when subjected to a pressure of 20Mpa. The bulk modulus for copper is 125000Mpa.

Solution

$\Delta p = 20MPa, B = 125000MPa, L = 40mm = 40 \times 10^{-3}m$

$$V = L^3 = (40 \times 10^{-3})^3 = 6.4 \times 10^{-5}m^3.$$

Using the formula

$$\Delta v = \frac{-v\Delta p}{B} = \frac{-(6.4 \times 10^{-5})(20 \times 10^6)}{125000 \times 10^6} = -1.0 \times 10^{-8}m^3 = 10mm^3.$$

Modulus of rigidity or shear modulus (G)

Shear stress: is defined as the force tangent to material surface divided by the area on which it acts.

Shear strain: is defined as the ratio of the displacement of the corner to the transverse dimension.

Provided Hooke's law is obeyed, the shear stress is directly proportional to shear strain and the elastic modulus in this case is called modulus of rigidity (G). or shear modulus or torsion modulus.

$$Modulus\ of\ rigidity\ G = \frac{shear\ stress}{shear\ strain} = \frac{F_{11}/A}{\Delta L/L}$$

Also $\frac{\Delta L}{L} = \tan\theta = \emptyset$

Solved examples

1. The shear modulus for a metal is 50MPa. Suppose that shear force of 200N is applied to the upper surface of a cube of this metal that is 3.0cm on each edge. How far will the top surface be displaced?

Solution

$G = 50 \times 10^6 Pa,\ F_{11} = 200N, L = 3.010^{-2}m,\ \Delta L = ?.$

$$\Delta L = \frac{F_{11}/A}{G/L} = \frac{200/(3.0\times10^{-2})^2}{5\times10^7/3.0\times10^{-2}} = 1.332 \times 10^{-4}m.$$

2. Two parallel and opposite forces, each 4000N, are applied tangentially to the upper and lower faces of a cubical metal block 25cm on a side. Find the angle of shear and the displacement of the upper surface relative to the lower surface. The shear modulus for the metal is 80GPa.

Solution

$F_{11} = 400N, G = 80 \times 10^9 Pa, L = 25 \times 10^{-2} m, A = L^2 = (25 \times 10^{-2})^2 = 6.25 \times 10^{-2} m^2.$

$$G = \frac{F_{11}}{A\emptyset}$$

$$\emptyset = \frac{F_{11}}{AG} = \frac{4000}{6.25 \times 10^{-2} \times 80 \times 10^9} = 8 \times 10^{-7} rad$$

Shear strain$\emptyset = \frac{\Delta L}{L}$,

$$\Delta L = L\emptyset = (25 \times 10^{-2})(8.0 \times 10^{-7}) = 2.0 \times 10^{-7} m.$$

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