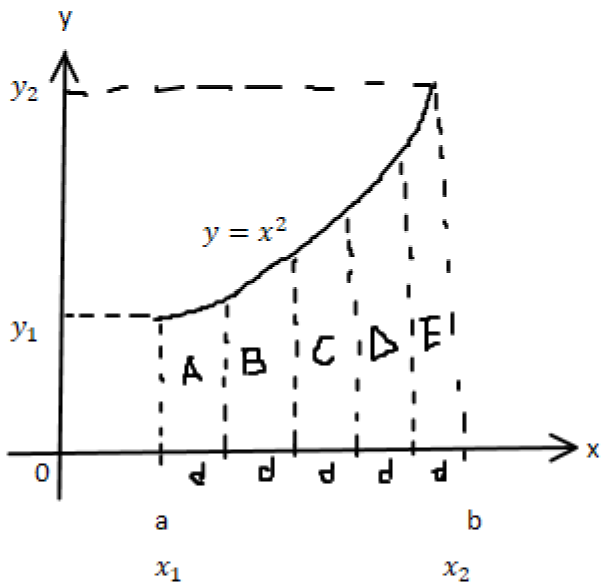
EGR3302 1st Summary

NUMERICAL INTEGRATION

Integration is no longer a mystery to you now, you have had several problems solved including the once you solved yourself. However, the one you know with its symbol ∫ is not the only approach that exist, there is Numerical Integration where instead of using the conventional $\int f(x)dx$, the function is represented on a graph, it is then divided into small equally spaced segments and the area of each segment is computed, the total is taken which is approximately the same as the integral of that function. This will be clear later.

Consider the figure below.



As you can see, there are five divisions between x_1 and x_2 each having a width of d. to find the integral.

$$y = \int_{x_1}^{x_2} x^2\, dx$$

All we have to do is find the individual areas of all the segment A, B, C…E and add them together i.e

$$x_1 = a \qquad x_2 = b$$

$$y = \int_{x_1=a}^{x_2=b} x^2\, dx = Area\ of\ (A + B + C + D + E)$$

Depending on how we compute the area, there are three methods of solving numerical integration.

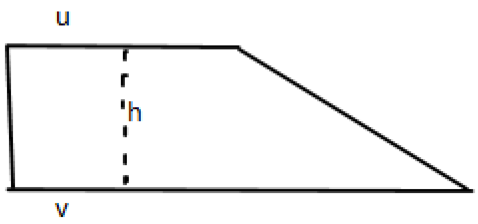
1. Trapezoidal rule
2. Mid ordinates rule

3. Simpsons rule.

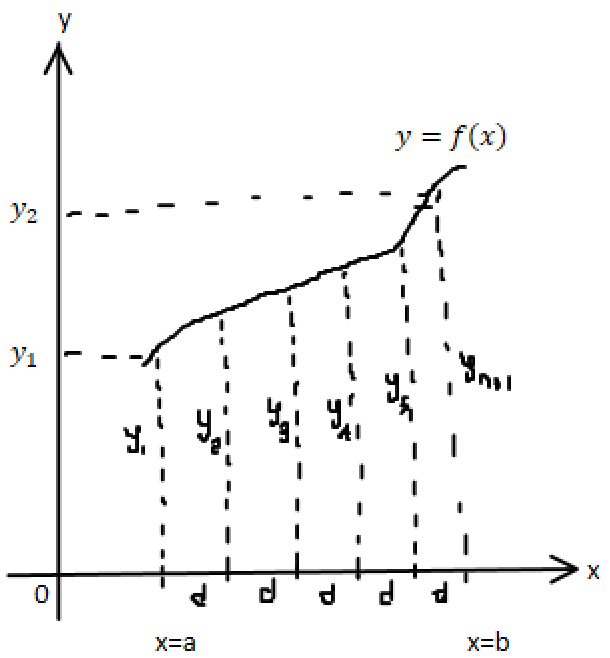
Trapezoidal Rule

The ideal of dividing the area under the curve to be integrated is used here.

Recall that trapezium, the area =$(u + v) \times {}^{h}/_{2}$



Now, consider the integral of the function $y = f(x)$ with limits of x=a and x=b, as represented below.



Where $d = \frac{b-a}{n}$

Since area of trapezium

$= \frac{1}{2}(sum\ of\ parallel\ sides) \times (perpendicular\ distance\ between\ them)$

$$\int_a^b y dx = \frac{1}{2}(y_1 + y_2)d + \frac{1}{2}(y_2 + y_3)d + \frac{1}{2}(y_3 + y_4)d + \frac{1}{2}(y_n + y_{n+1})d + \cdots$$

$$=\frac{d}{2}(y_1 + y_2 + y_2 + y_3 + y_3 + y_4 + y_n + y_{n+1})$$

$y_n + y_{n+1}$ Appeared once

$$\frac{d}{2}(y_1 + y_{n+1} + 2y_2 + 2y_3 + 2y_4 + \cdots 2y_n)$$

$$d\left(\frac{1}{2}(y_1 + y_{n+1}) + (y_2 + y_3 + y_4 + \cdots y_n)\right)$$

Example1.

Evaluate $y = \int_1^2 x^2\, dx$

1. Using Trapezoidal rule
    a. Using four intervals
    b. Using eight intervals correct to 4d.p

<u>Solution</u>

a.

Using Trapezoidal rule n = 4

<u>d</u>= $\frac{b-a}{n} = \frac{2-1}{4} = 0.25$

The ordinates occurs at

x= 1.00, 1.25, 1.50, 1.75, 2.00

X	1.00	1.25	1.50	1.75	2.00
Y= x^2	1.00	1.5625	2.250	3.0625	4.00
	y_1	y_2	y_3	y_4	y_5

$\int_1^2 x^2\, dx$ = d $\left(\frac{1}{2}(y_1 + y_5) + (y_2 + y_3 + y_4)\right)$

= $0.25 \left(\frac{1}{2}(1 + 4) + (1.5625 + 2.250 + 3.0625)\right)$

= 2.34375

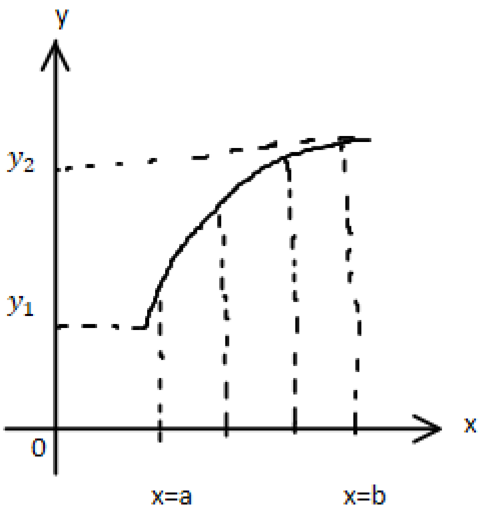
≈2.3438

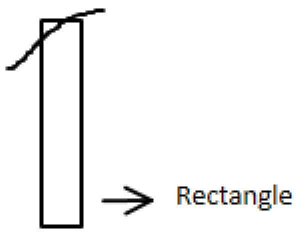
The actual integration would be $\left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = 2.333$

b) d_o For n = 8

Mid Ordinate Rule

If we consider an integral $\int_a^b ydx$ with the limits $x = b$ as represented below.





In this instance, each area is approximated to a rectangle as the middle of the ordinate. Hence area the total area will be

$$\int_a^b ydx = d_y + d_{y2} + d_{y3} + \cdots d_{yn}$$

$$\approx d\{y_1 + y_2 + y_3 + \cdots y_n\}$$

= {width of the interval} {sum of mid-ordinates}

Evaluate $\int_1^2 x^2 dx$ using

2. Using mid ordinate rule
   a. Using eight intervals correct to 4d.p
   b. Using four intervals

<u>Solution</u>

b. n=4

$\underline{d} = \frac{b-a}{n} = \frac{2-1}{4} = 0.25$

The ordinates occurs at

X = 1.00, 1.25, 1.50, 1.75, 2.00

Mid-ordinate will occur at

x=1.125, 1.375, 1.625 and 1.875

Hint,

$$\frac{1.00+1.25}{2} = 1.125$$

$$\frac{1.25+1.50}{2} = 1.375$$

And so 0.1

x	1.125	1.375	1.625	1.875
y	1.2656	1.8906	2.6406	3.5156

$$\int_1^2 x^2 dx = 0.25\{1.2656 + 1.8906 + 2.6406 + 3.5156\}$$

= 2.3281

Checking the most accurate

Trapezoidal value	2.3438	0.45%
Mid ordinate value	2.3281	0.22%
Actual value	2.3333	-

$$\%error = \frac{Actual - Numerical}{Actual} \times 100\%$$

Trap $\%error = \frac{2.3333-2.3438}{2.3333} \times 100\% = 0.45\%$

Mid ord, $\%error = \frac{2.3333-2.3281}{2.3333} \times 100\% = 0.22\%$

Therefore Mid ordinate is more accurate.

Example2. Evaluate $\int_0^1 \sqrt{(2x+1)}\partial x$ at an interval of 4 using Trapezoidal rule.

<u>Solution</u>

a=0, b=1, n=4.

$$d = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

Therefore ordinate will occur at

X= 0, 0.25, 0.50, 0.75 and 1.00

(Addition of d=0.25 at regular interval starting from the initial point “a”. The last ordinate will exactly be the same a “b”)

$$y = \sqrt{(2x+1)}$$

X	0.00	0.25	0.50	0.75	1.00
y	1.0000	1.2247	1.4142	1.5811	1.7321

$$I = \int_a^b y\partial x = \int_0^1 \sqrt{(2x+1)}\partial x$$

Using trapezoidal rule

Where y_0 & y_n are the first and last term respectively.

$$I = d\left[\frac{y_0 + y_n}{2} + sum\ of\ the\ other\ ordinates\right]$$

$$I = 0.25\left[\frac{1+1.732}{2} + 1.2247 + 1.4142 + 1.5811\right] \approx 1.3965$$

NB: for trapezoidal rule, number of ordinates is always equal to interval+1 i.e. (1+n).

Example3.

Evaluate $\int_0^1 \cos(x^3 + x)\partial x$ using four intervals using the Trapezoidal rule.

<u>Solution</u>

a=0, b=1 n=4

$$d = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

Therefore ordinate will occur at 0, 0.25, 0.50, 0.75 and 1.00

$y = \cos(x^3 + x)$ To compute this, your calculator must be in radian

X	0.00	0.25	0.50	0.75	1.00
y	1.0000	0.9649	0.8109	0.3884	-0.4161

$$I = 0.25\left[\frac{1-.4161}{2} + 0.9649 + 0.8109 + 0.3884\right] \approx 0.6140$$

NB: whenever you are dealing with trigonometric function such as sine, cos, sinh and tan. Always put your calculator in **radian** if the corresponding value of x is not in degree.

Example4.

Evaluate $\int_0^1 \sqrt{(2x+1)}\partial x$ at an interval of 4 using Mid-ordinate rule

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<u>Solution</u>

a=0, b=1, n=4.

$$d = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

Therefore ordinate will occur at 0, 0.25, 0.50, 0.75 and 1.00

The mid ordinates are; 0.125, 0.375, 0.625 and 0.875.

X	0.1250	0.3750	0.6250	0.8750
Y	1.1180	1.3229	1.5000	1.6583

$$I = 0.25(1.1180 + 1.3229 + 1.5000 + 1.6583) \approx 1.3998$$

Example5.

Evaluate $\int_0^1 \cos(x^3 + x)\partial x$ using four intervals using Mid-ordinate rule.

Solution

a=0, b=1 n=4

$$d = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

Therefore ordinate will occur at 0, 0.25, 0.50, 0.75 and 1.00

X	0.1250	0.3750	0.6250	0.8750
Y	0.9919	0.9099	0.6455	0.0259

$$I = 0.25(0.9919 + 0.9099 + 0.6455 + 0.0259) \approx 0.6432$$

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