EGR2302 1st Summary

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Vector Differential Operator$(del\nabla)$

$\nabla = \frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}$

∇ Is useful in defining three parameters which arise in practical application are known as gradient, divergence and the curl. The operator $\nabla$is also called $(nebla)$

The gradient;

Let $\emptyset(x, y, z)$ be defined and differentiable each point (x, y, z) in a Cartesian region of space the gradient of $\emptyset$ is written as $\nabla\emptyset(neblaphi, delphi\ or\ phi)$ which is defined as;

$\nabla\emptyset = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right)\emptyset$

$\frac{d\emptyset}{dx}i + \frac{d\emptyset}{dy}j + \frac{d\emptyset}{dz}k$

The component of $\nabla\emptyset \cdot \vec{n}$ and is called the directional derivative of $\emptyset$ in the direction of unit vector$\vec{n}$.

The Divergence

Let $\vec{v}(x, y, z) = v_1 i + v_2 j + v_3 k$ be defined and differentiable at each point (x, y, z)in a certain region of space (v defines differentiable vector field) then the divergence of $\vec{v}$ written as $\nabla \cdot \vec{v}\ or\ div\ \vec{v}$

$\nabla \cdot \vec{v} = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right) \cdot (v_1 i + v_2 j + v_3 k\ )$

$= \frac{dv_1}{dx} + \frac{dv_2}{dy} + \frac{dv_3}{dz}$

NOTE; $\nabla \cdot \overrightarrow{v} \quad \vec{v} \cdot \nabla$

The Curl

If $v(x, y, z)$ is a differentiable vector field, then the curl or rotation of $\overrightarrow{v}$ is written as;

$$\nabla \times \vec{v} \; or \; curl \, \overrightarrow{v} \; or \; rot \, \overrightarrow{v} =$$

$$\nabla \times \vec{v} = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k\right) \cdot (v_1 i + v_2 j + v_3 k\ ) =$$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ v_1 & v_2 & v_3 \end{vmatrix} = i\left(\frac{dv_3}{dy} - \frac{dv_2}{dz}\right) - j\left(\frac{dv_3}{dx} - \frac{dv_1}{dz}\right) + k\left(\frac{dv_2}{dx} - \frac{dv_1}{dy}\right)$$

In summary

a. Gradient is multiplication of ∇ and a scalar quantity.
b. Divergence is the dot production of ∇ and a vector quantity.
c. Curl is the cross product of ∇ and vector quantity.

Where ∇ is called ''Del'' or ''nebla'' of it representation

$$\left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k\right)$$

Examples

1. If $\emptyset\ (x, y, z) = 3x^2y - y^3z^2$, find $\nabla\emptyset$ at $(1, -2, -1)$

Soln

$$\nabla\emptyset = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k\right)\emptyset = \left(\frac{d\emptyset}{dx} i + \frac{d\emptyset}{dy} j + \frac{d\emptyset}{dz} k\right)$$

$$\frac{d\emptyset}{dx} = \frac{d}{dx}(3x^2y - y^3z^2) = 6xy$$

$$\frac{d\emptyset}{dy} = \frac{d}{dy}(3x^2y - y^3z^2) = 3x^2 - 3y^2z^2$$

$$\frac{d\emptyset}{dz} = \frac{d}{dz}(3x^2y - y^3z^2) = -2zy^3$$

$\nabla\emptyset = (6xy)i + (3x^2 - 3y^2z^2)j - 2zy^3k$

$\nabla\emptyset_{(1,-2,-1)} = 6(1)(-2)i + \left(3(1^2) - 3(-2^2)(1^2)\right)j - 2(-1)(-2^3)k$

$\nabla\emptyset_{(1,-2,-1)} = -12i - 9j - 16k$

$$\frac{df(x)}{dx} = \frac{f(x)}{f(x)} \qquad \frac{dlnf(x)}{dx} = \frac{f'(x)}{f(n)}$$

2. Find$\nabla\emptyset$, if
   a. $\emptyset = ln|\vec{r}|$
   b. $\emptyset = \frac{1}{r}$

Given $\vec{r} = (xi + yj + zk)$

Soln

$$\vec{r} = (xi + yj + zk)$$

a. $\emptyset = ln|\vec{r}| = ln\left(\sqrt{x^2 + y^2 + z^2}\right) = ln[x^2 + y^2 + z^2]^{\frac{1}{2}}$

$\emptyset = \frac{1}{2} ln[x^2 + y^2 + z^2]$

$\nabla\emptyset \left(\frac{d\emptyset}{dx}i + \frac{d\emptyset}{dy}j + \frac{d\emptyset}{dz}k\right)$

$\frac{d\emptyset}{dx} = \frac{d}{dx}\left[\frac{1}{2} ln[x^2 + y^2 + z^2]\right] = \frac{1}{2} \cdot \frac{d}{dx}\left[ln[x^2 + y^2 + z^2]\right]$

$\frac{d\emptyset}{dx} = \frac{1}{2} \cdot \frac{2x}{x^2+y^2+z^2} = \qquad \frac{x}{x^2+y^2+z^2}$

$\frac{d\emptyset}{dy} = \frac{d}{dy}\left[\frac{1}{2} ln[x^2 + y^2 + z^2]\right] = \frac{1}{2} \cdot \frac{d}{dy}\left[ln[x^2 + y^2 + z^2]\right]$

$\frac{d\emptyset}{dy} = \frac{1}{2} \cdot \frac{2y}{x^2+y^2+z^2} = \qquad \frac{y}{x^2+y^2+z^2}$

$\frac{d\emptyset}{dz} = \frac{d}{dz}\left[\frac{1}{2} ln[x^2 + y^2 + z^2]\right] = \frac{1}{2} \cdot \frac{d}{dz}\left[ln[x^2 + y^2 + z^2]\right]$

$\frac{d\emptyset}{dz} = \frac{1}{2} \cdot \frac{2z}{x^2+y^2+z^2} = \qquad \frac{z}{x^2+y^2+z^2}$

$= \nabla\emptyset = \left(\frac{x}{x^2+y^2+z^2}\right)i + \left(\frac{y}{x^2+y^2+z^2}\right)j + \left(\frac{z}{x^2+y^2+z^2}\right)k$

$$\frac{df(x)^n}{dx} = n.f'(x) \cdot f(x)^{n-1}$$

b. $\emptyset = \frac{1}{r} = r^{-1} = [x^2 + y^2 + z^2]^{-\frac{1}{2}}$

$\nabla\emptyset = \left(\frac{d\emptyset}{dx}i + \frac{d\emptyset}{dy}j + \frac{d\emptyset}{dz}k\right)$

$\frac{d\emptyset}{dx} = \frac{d}{dx}[x^2 + y^2 + z^2]^{-\frac{1}{2}} \quad = -\frac{1}{2} \cdot 2x[x^2 + y^2 + z^2]^{-\frac{3}{2}}$

$= \frac{-x}{[x^2+y^2+z^2]^{-\frac{3}{2}}}$

$\frac{d\emptyset}{dy} = \frac{d}{dy}[x^2 + y^2 + z^2]^{-\frac{1}{2}} \quad = -\frac{1}{2} \cdot 2y[x^2 + y^2 + z^2]^{-\frac{3}{2}}$

$= \frac{-y}{[x^2+y^2+z^2]^{-\frac{3}{2}}}$

$\frac{d\emptyset}{dz} = \frac{d}{dz}[x^2 + y^2 + z^2]^{-\frac{1}{2}} \quad = -\frac{1}{2} \cdot 2z[x^2 + y^2 + z^2]^{-\frac{3}{2}}$

$= \frac{-z}{[x^2+y^2+z^2]^{-\frac{3}{2}}}$

$= \nabla\emptyset = \left(\frac{-xi}{[x^2+y^2+z^2]^{-\frac{3}{2}}}\right) - \left(\frac{yj}{[x^2+y^2+z^2]^{-\frac{3}{2}}}\right) - \left(\frac{zk}{x^2+y^2+z^2}\right)$

$\frac{(xi+yj+zk)}{[x^2+y^2+z^2]^{\frac{3}{2}}} = \frac{-r}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$

$\frac{-r}{(|\vec{r}|)^3}$

3. $If \ \vec{A} = x^3yi + 2y^2zj - 2xz^2k \quad find \ \nabla.\vec{A} \ \ a(2, -1, 1)$

Soln

$\nabla.\vec{A} = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right).(x^3yi + 2y^2zj - xy^2z^2k)$

$\nabla.\vec{A} = \frac{d}{dx}(x^3y) + \frac{d}{dy}(2y^2z) - \frac{d}{dz}(xy^2z^2)$

$= 3x^2y + 4yz - 2xy^2z$

$\nabla.\vec{A}_{(2,-1,1)} = 3(2)^2(-1) + 4(-1)(1) - 2(2)(-1)^2(1)$

$= -12 - 4 - 4 = -20$

4. if $\emptyset = 2x^2y^3z^2 \quad find \ \nabla.\nabla\emptyset$

Soln

Let’s first evaluate the gradient i.e. $\nabla\emptyset$

$\nabla\emptyset = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right)\emptyset = \frac{d\emptyset}{dx}i + \frac{d\emptyset}{dy}j + \frac{d\emptyset}{dz}k$

$\frac{d\emptyset}{dx}i = \frac{d}{dx}(2x^2y^3z^2)i = 4xy^3z^2i$

$\frac{d\emptyset}{dy}j = \frac{d}{dy}(2x^2y^3z^2)j = (3y^2.2x^2z^2)j = 6x^2y^2z^2j$

$\frac{d\emptyset}{dz}k = \frac{d}{dz}(2x^2y^3z^2)k = 4x^2y^3zk$

$= 4xy^3z^2i + 6x^2y^2z^2j + 4x^2y^3zk$

<u>*now*</u>

$\nabla.\nabla\emptyset = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right).(4xy^3z^2i + 6x^2y^2z^2j + 4x^2y^3zk)$

$= \frac{d}{dx}(4xy^3z^2) + \frac{d}{dy}(6x^2y^2z^2) + \frac{d}{dz}(4x^2y^3z)$

$= 4y^3z + 12x^2yz^2 + 4x^2y^3$

5. Given that $\vec{A} = 2xz^2i - x^2yzj + 2yz^2k, \quad find\ \nabla \times \vec{A} \quad c(2,-2,2)$

<u>Soln</u>

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \dfrac{d}{dx} & \dfrac{d}{dy} & \dfrac{d}{dz} \\ 2xz^2 & -x^2yz & 2yz^2 \end{vmatrix}$$

$\left(\frac{d}{dy}(2yz^2) - \frac{d}{dz}(-x^2yz)k\right)i - \left(\frac{d}{dx}(2yz^2) - \frac{d}{dz}(2xz^2)\right)j + \left(\frac{d}{dx}(-x^2yz) - \frac{d}{dy}(2xz^2)\right)k \quad = i(2z^2 + x^2y) - j(0 - 4xz) + k(-2xyz - 0)$

$= (2z^2 + x^2y)i + 4xzj - 2xyzk \qquad at(2,-2,2)$

$\nabla \times \vec{A}_{(2,-2,2)} = \left(2(2)^2 + (2)^2(-2)\right)i + 4(2)(2)j - 2(2)(-2)(2)k$

$= (8-8)i + 16j + 16k$

$= 16j + 16k$

6. Given that $\vec{A} = 2xz^2i - x^2yzj + 2yz^2k,\ find\ \ \nabla \times \emptyset$

<u>Soln</u>

$$\nabla \times \emptyset \ = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 3x^2y & -y^3z^2 & 2xz^2 \end{vmatrix}$$

$$= \left(\frac{d}{dy}(2xz^2) + \frac{d}{dz}(y^3z^2)\right)i - \left(\frac{d}{dx}(2xz^2) - \frac{d}{dz}(3x^2)\right)\text{j}+\left(\frac{d}{dx}(-y^3z^2) - \frac{d}{dy}(3x^2y)\right)\text{k}$$

$$= (0 + 2y^3z)i - (2z^2 - 0)j + (0 - 3x^2)k$$

$$= 2y^3zi - 2z^2j - 3x^2k$$

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